**Toothed Gearing**

**Introduction:**

We have discussed in the previous chapter, that the slipping of a belt or rope is a common phenomenon, in the transmission of motion or power between two shafts. The effect of slipping is to reduce the velocity ratio of the system. In precision machines, in which a definite velocity ratio is of importance (as in watch mechanism), the only positive drive is by means of _gears_ or _toothed wheels_. A gear drive is also provided, when the distance between the driver and the follower is very small.

**Friction Wheels:**

The motion and power transmitted by gears is kinematically equivalent to that transmitted by friction wheels or discs. In order to understand how the motion can be transmitted by two toothed wheels, consider two plain circular wheels A and B mounted on shafts, having sufficient rough surfaces and pressing against each other as shown in Fig. 12.1 (a).

Let the wheel A be keyed to the rotating shaft and the wheel B to the shaft, to be rotated. A little consideration will show (that when the wheel A is rotated by a rotating shaft) it will rotate the wheel B in the opposite direction as shown in Fig. 12.1 (a).

The wheel B will be rotated (by the wheel A) so long as the tangential force exerted by the wheel A does not exceed the maximum frictional resistance between the two wheels. But when the tangential force (P) exceeds the frictional resistance (F), slipping will take place between the two wheels. Thus the friction drive is not a positive drive.

![Fig. 12.1](image)

In order to avoid the slipping, a number of projections (called teeth) as shown in Fig. 12.1 (b), are provided on the periphery of the wheel A, which will fit into the corresponding recesses on the periphery of the wheel B. A friction wheel with the teeth cut on it is known as _toothed wheel_ or _gear_. The usual connection to show the toothed wheels is by their pitch circles.
Note: Kinematically, the friction wheels running without slip and toothed gearing are identical. But due to the possibility of slipping of wheels, the friction wheels can only be used for transmission of small powers.

Advantages and Disadvantages of Gear Drive:

The following are the advantages and disadvantages of the gear drive as compared to belt, rope and chain drives:

Advantages
1. It transmits exact velocity ratio.
2. It may be used to transmit large power.
3. It has high efficiency.
4. It has reliable service.
5. It has compact layout.

Disadvantages
1. The manufacture of gears require special tools and equipment.
2. The error in cutting teeth may cause vibrations and noise during operation.

Classification of Toothed Wheels:

The gears or toothed wheels may be classified as follows:

1. According to the position of axes of the shafts. The axes of the two shafts between which the motion is to be transmitted, may be
   (a) Parallel, (b) Intersecting, and (c) Non-intersecting and non-parallel.

The two parallel and co-planar shafts connected by the gears is shown in Fig. 12.1. These gears are called spur gears and the arrangement is known as spur gearing. These gears have teeth parallel to the axis of the wheel as shown in Fig. 12.1. Another name given to the spur gearing is helical gearing, in which the teeth are inclined to the axis. The single and double helical gears connecting parallel shafts are shown in Fig. 12.2 (a) and (b) respectively. The double helical gears are known as herringbone gears. A pair of spur gears is kinematically equivalent to a pair of cylindrical discs, keyed to parallel shafts and having a line contact.

The two non-parallel or intersecting, but coplanar shafts connected by gears is shown in Fig. 12.2 (c). These gears are called bevel gears and the arrangement is known as bevel gearing. The bevel gears, like spur gears, may also have their teeth inclined to the face of the bevel, in which case they are known as helical bevel gears.

The two non-intersecting and non-parallel i.e. non-coplanar shaft connected by gears is shown in Fig. 12.2 (d). These gears are called skew bevel gears or spiral gears and the arrangement is known as skew bevel gearing or spiral gearing. This type of gearing also have a line contact, the rotation of which about the axes generates the two pitch surfaces known as hyperboloids.

Notes: (a) When equal bevel gears (having equal teeth) connect two shafts whose axes are mutually perpendicular, then the bevel gears are known as mitres.

(b) A hyperboloid is the solid formed by revolving a straight line about an axis (not in the same plane), such that every point on the line remains at a constant distance from the axis.

(c) The worm gearing is essentially a form of spiral gearing in which the shafts are usually at right angles.
2. **According to the peripheral velocity of the gears.** The gears, according to the peripheral velocity of the gears may be classified as:

(a) Low velocity, (b) Medium velocity, and (c) High velocity.

The gears having velocity less than 3 m/s are termed as low velocity gears and gears having velocity between 3 and 15 m/s are known as medium velocity gears. If the velocity of gears is more than 15 m/s, then these are called high speed gears.

3. **According to the type of gearing.** The gears, according to the type of gearing may be classified as:

(a) External gearing, (b) Internal gearing, and (c) Rack and pinion.

In **external gearing**, the gears of the two shafts mesh externally with each other as shown in Fig.12.3 (a). The larger of these two wheels is called spur wheel and the smaller wheel is called pinion. In an external gearing, the motion of the two wheels is always unlike, as shown in Fig. 12.3 (a).

In **internal gearing**, the gears of the two shafts mesh internally with each other as shown in Fig. 12.3 (b). The larger of these two wheels is called annular wheel and the smaller wheel is called pinion. In an internal gearing, the motion of the two wheels is always like, as shown in Fig. 12.3 (b).

Sometimes, the gear of a shaft meshes externally and internally with the gears in a straight line, as shown in Fig. 12.4. Such type of gear is called **rack and pinion**. The straight line gear is called rack and the circular wheel is called pinion. A little consideration will show that with the help of a rack and pinion, we can convert linear motion into rotary motion and vice-versa as shown in Fig.12.4.
4. According to position of teeth on the gear surface. The teeth on the gear surface may be (a) straight, (b) inclined, and (c) curved.

We have discussed earlier that the spur gears have straight teeth where as helical gears have their teeth inclined to the wheel rim. In case of spiral gears, the teeth are curved over the rim surface.

**Terms Used in Gears:**

The following terms, which will be mostly used in this chapter, should be clearly understood at this stage. These terms are illustrated in Fig. 12.5.

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Fig. 12.5. Terms used in gears.

1. **Pitch circle.** It is an imaginary circle which by pure rolling action, would give the same motion as the actual gear.

2. **Pitch circle diameter.** It is the diameter of the pitch circle. The size of the gear is usually specified by the pitch circle diameter. It is also known as pitch diameter.

3. **Pitch point.** It is a common point of contact between two pitch circles.

4. **Pitch surface.** It is the surface of the rolling discs which the meshing gears have replaced at the pitch circle.

5. **Pressure angle or angle of obliquity.** It is the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitch point. It is usually denoted by $\phi$. The standard pressure angles are 14.5° and 20°.

6. **Addendum.** It is the radial distance of a tooth from the pitch circle to the top of the tooth.

7. **Dedendum.** It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.

8. **Addendum circle.** It is the circle drawn through the top of the teeth and is concentric with the pitch circle.

9. **Dedendum circle.** It is the circle drawn through the bottom of the teeth. It is also called root circle.

*Note: Root circle diameter = Pitch circle diameter $\times$ cos $\phi$, where $\phi$ is the pressure angle.*
10. **Circular pitch.** It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth. It is usually denoted by \( p_c \).

Mathematically,

Circular pitch, \( p_c = \pi \frac{D}{T} \)

Where \( D \) = Diameter of the pitch circle, and \( T \) = Number of teeth on the wheel.

A little consideration will show that the two gears will mesh together correctly, if the two wheels have the same circular pitch.

**Note:** If \( D_1 \) and \( D_2 \) are the diameters of the two meshing gears having the teeth \( T_1 \) and \( T_2 \) respectively, then for them to mesh correctly,

\[
p_c = \frac{\pi D_1}{T_1} = \frac{\pi D_2}{T_2} \quad \text{or} \quad \frac{D_1}{D_2} = \frac{T_1}{T_2}
\]

11. **Diametral pitch.** It is the ratio of number of teeth to the pitch circle diameter in millimeters. It is denoted by \( p_d \). Mathematically, Diametral pitch is

\[
p_d = \frac{T}{D} = \frac{\pi}{p_c} \quad \therefore \quad p_c = \pi \frac{D}{T}
\]

Where \( T \) = Number of teeth, and \( D \) = Pitch circle diameter.

12. **Module.** It is the ratio of the pitch circle diameter in millimeters to the number of teeth. It is usually denoted by \( m \). Mathematically, Module, \( m = \frac{D}{T} \).

**Note:** The recommended series of modules in Indian Standard are 1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, and 20. The modules 1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14 and 18 are of second choice.

13. **Clearance.** It is the radial distance from the top of the tooth to the bottom of the tooth, in a meshing gear. A circle passing through the top of the meshing gear is known as clearance circle.

14. **Total depth.** It is the radial distance between the addendum and the dedendum circles of a gear. It is equal to the sum of the addendum and dedendum.

15. **Working depth.** It is the radial distance from the addendum circle to the clearance circle. It is equal to the sum of the addendum of the two meshing gears.

16. **Tooth thickness.** It is the width of the tooth measured along the pitch circle.

17. **Tooth space.** It is the width of space between the two adjacent teeth measured along the pitch circle.

18. **Backlash.** It is the difference between the tooth space and the tooth thickness, as measured along the pitch circle. Theoretically, the backlash should be zero, but in actual practice some backlash must be allowed to prevent jamming of the teeth due to tooth errors and thermal expansion.

19. **Face of tooth.** It is the surface of the gear tooth above the pitch surface.

20. **Flank of tooth.** It is the surface of the gear tooth below the pitch surface.

21. **Top land.** It is the surface of the top of the tooth.

22. **Face width.** It is the width of the gear tooth measured parallel to its axis.

23. **Profile.** It is the curve formed by the face and flank of the tooth.

24. **Fillet radius.** It is the radius that connects the root circle to the profile of the tooth.
25. **Path of contact.** It is the path traced by the point of contact of two teeth from the beginning to the end of engagement.

26. **Length of the path of contact.** It is the length of the common normal cut-off by the addendum circles of the wheel and pinion.

27. **Arc of contact.** It is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth. The arc of contact consists of two parts, i.e.

   (a) **Arc of approach.** It is the portion of the path of contact from the beginning of the engagement to the pitch point.

   (b) **Arc of recess.** It is the portion of the path of contact from the pitch point to the end of the engagement of a pair of teeth.

   **Note:** The ratio of the length of arc of contact to the circular pitch is known as contact ratio i.e. number of pairs of teeth in contact.

**Gear Materials:**

The material used for the manufacture of gears depends upon the strength and service conditions like wear, noise etc. The gears may be manufactured from metallic or non-metallic materials. The metallic gears with cut teeth are commercially obtainable in cast iron, steel and bronze. The nonmetallic materials like wood, raw hide, compressed paper and synthetic resins like nylon are used for gears, especially for reducing noise. The cast iron is widely used for the manufacture of gears due to its good wearing properties, excellent machinability and ease of producing complicated shapes by casting method.

The cast iron gears with cut teeth may be employed, where smooth action is not important.

The steel is used for high strength gears and steel may be plain carbon steel or alloy steel. The steel gears are usually heat treated in order to combine properly the toughness and tooth hardness.

The phosphor bronze is widely used for worm gears in order to reduce wear of the worms which will be excessive with cast iron or steel.

**Condition for Constant Velocity Ratio of Toothed Wheels—Law of Gearing:**

Consider the portions of the two teeth, one on the wheel 1 (or pinion) and the other on the wheel 2, as shown by thick line curves in Fig. 12.6. Let the two teeth come in contact at point $Q$, and the wheels rotate in the directions as shown in the figure.

Let $TT$ be the common tangent and $MN$ be the common normal to the curves at the point of contact $Q$. From the centers $O_1$ and $O_2$, draw $O_1M$ and $O_2N$ perpendicular to $MN$. A little consideration will show that the point $Q$ moves in the direction $QC$, when considered as a point on wheel 1, and in the direction $QD$ when considered as a point on wheel 2.

Let $v_1$ and $v_2$ be the velocities of the point $Q$ on the wheels 1 and 2 respectively. If the teeth are to remain in contact, then the components of these velocities along the common normal $MN$ must be equal.

\[
\therefore \quad v_1 \cos \alpha = v_2 \cos \beta
\]

or

\[
(\omega_1 \times O_1 Q) \cos \alpha = (\omega_2 \times O_2 Q) \cos \beta
\]
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Fig. 12.6. Law of gearing.

Also from similar triangles \( O_1MP \) and \( O_2NP \),

\[
\frac{O_1N}{O_1M} = \frac{O_2P}{O_2M} \quad \text{...}(i)
\]

Combining equations \((i)\) and \((ii)\), we have

\[
\frac{\omega_1}{\omega_2} = \frac{O_1N}{O_1M} = \frac{O_2P}{O_2M} \quad \text{...}(ii)
\]

From above, we see that the angular velocity ratio is inversely proportional to the ratio of the distances of the point \( P \) from the centres \( O_1 \) and \( O_2 \), or the common normal to the two surfaces at the point of contact \( Q \) intersects the line of centres at point \( P \) which divides the centre distance inversely as the ratio of angular velocities.

Therefore in order to have a constant angular velocity ratio for all positions of the wheels, the point \( P \) must be the fixed point (called pitch point) for the two wheels. In other words, **the common normal at the point of contact between a pair of teeth must always pass through the pitch point.**

This is the fundamental condition which must be satisfied while designing the profiles for the teeth of gear wheels. It is also known as **law of gearing.**

**Notes:**

1. The above condition is fulfilled by teeth of involute form, provided that the root circles from which the profiles are generated are tangential to the common normal.

2. If the shape of one tooth profile is arbitrarily chosen and another tooth is designed to satisfy the above condition, then the second tooth is said to be conjugate to the first. The conjugate teeth are not in common use because of difficulty in manufacture, and cost of production.

3. If \( D_1 \) and \( D_2 \) are pitch circle diameters of wheels 1 and 2 having teeth \( T_1 \) and \( T_2 \) respectively, then velocity ratio,

\[
\frac{\omega_1}{\omega_2} = \frac{O_1P}{O_1M} = \frac{D_2}{D_1} = \frac{T_2}{T_1}
\]
**Velocity of Sliding of Teeth:**

The sliding between a pair of teeth in contact at $Q$ occurs along the common tangent $TT$ to the tooth curves as shown in Fig. 12.6. The velocity of sliding is the velocity of one tooth relative to its mating tooth along the common tangent at the point of contact.

The velocity of point $Q$, considered as a point on wheel 1, along the common tangent $TT$ is represented by $EC$. From similar triangles $QEC$ and $O_1MQ$,

$$\frac{EC}{MQ} = \frac{v}{OQ} = \omega_1 \quad \text{or} \quad EC = \omega_1 \cdot MQ$$

Similarly, the velocity of point $Q$, considered as a point on wheel 2, along the common tangent $TT$ is represented by $ED$. From similar triangles $QCD$ and $O_2NQ$,

$$\frac{ED}{QN} = \frac{v_2}{O_2Q} = \omega_2 \quad \text{or} \quad ED = \omega_2 \cdot QN$$

Let $v_S = \text{Velocity of sliding at } Q$.

$$v_S = ED - EC = \omega_2 \cdot QN - \omega_1 \cdot MQ = \omega_2 \cdot (QP + PN) - \omega_1 \cdot (MP - QP) = (\omega_1 + \omega_2) \cdot QP + \omega_2 \cdot PN - \omega_1 \cdot MP \quad \text{(i)}$$

Since $\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P} = \frac{PN}{QP}$ or $\omega_1 \cdot MP = \omega_2 \cdot PN$, therefore equation (i) becomes

$$v_S = (\omega_1 + \omega_2) \cdot QP \quad \text{(ii)}$$

**Notes:**

1. We see from equation (ii), that the velocity of sliding is proportional to the distance of the point of contact from the pitch point.

2. Since the angular velocity of wheel 2 relative to wheel 1 is $(\omega_1 + \omega_2)$ and $P$ is the instantaneous centre for this relative motion, therefore the value of $v_S$ may directly be written as $v_S \cdot (\omega_1 + \omega_2) \cdot QP$, without the above analysis.

**Involute Teeth:**

An involute of a circle is a plane curve generated by a point on a tangent, which rolls on the circle without slipping or by a point on a taut string which in unwrapped from a reel as shown in Fig. 12.9. In connection with toothed wheels, the circle is known as base circle. The involute is traced as follows:

Let $A$ be the starting point of the involute. The base circle is divided into equal number of parts e.g. $AP_1$, $P_1P_2$, $P_2P_3$ etc. The tangents at $P_1$, $P_2$, $P_3$ etc. are drawn and the length $P_1A_1$, $P_2A_2$, $P_3A_3$ equal to the arcs $AP_1$, $AP_2$ and $AP_3$ are set off. Joining the points $A$, $A_1$, $A_2$, $A_3$ etc. we obtain the involute curve $AR$. A little consideration will show that at any instant $A_3$, the tangent $A_3T$ to the involute is perpendicular to $P_3A_3$ and $P_3A_3$ is the normal to the involute. In other words, normal at any point of an involute is a tangent to the circle.

Now, let $O_1$ and $O_2$ be the fixed centres of the two base circles as shown in Fig. 12.10 (a). Let the corresponding involutes $AB$ and $A_1B_1$ be in contact at point $Q$. $MQ$ and $NQ$ are normal to the involutes at $Q$ and are tangents to base circles. Since the normal of an involute at a given point is the tangent drawn from that point to the base circle, therefore the common normal $MN$ at $Q$ is also the common tangent to the two base circles. We see that the common normal $MN$ intersects the line of centers $O_1O_2$ at the fixed point $P$ (called pitch point). Therefore the involute teeth satisfy the fundamental condition of constant velocity ratio.
Fig. 12.10. Involute teeth.

From similar triangles \( O_2NP \) and \( O_1MP \),

\[
\frac{Q_2M}{Q_2N} = \frac{Q_1P}{Q_1P} = \frac{o_2}{o_1}
\]

which determines the ratio of the radii of the two base circles. The radii of the base circles is given by \( O_1M = O_1P \cos \phi \), and \( O_2N = O_2P \cos \phi \).

Also the centre distance between the base circles,

\[
QO_1' = Q_1P + O_1P = \frac{Q_1M}{\cos \phi} + \frac{Q_2N}{\cos \phi} = \frac{Q_1M + O_1N}{\cos \phi}
\]

where \( \phi \) is the pressure angle or the angle of obliquity. It is the angle which the common normal to the base circles (i.e. \( MN \)) makes with the common tangent to the pitch circles. When the power is being transmitted, the maximum tooth pressure (neglecting friction at the teeth) is exerted along the common normal through the pitch point. This force may be resolved into tangential and radial or normal components. These components act along and at right angles to the common tangent to the pitch circles.

If \( F \) is the maximum tooth pressure as shown in Fig. 12.10 (b), then

Tangential force, \( F_T = F \cos \phi \)

and radial or normal force, \( F_R = F \sin \phi \).

\[ \therefore \text{Torque exerted on the gear shaft} = F_T \times r, \text{ where } r \text{ is the pitch circle radius of the gear.} \]

**Note:** The tangential force provides the driving torque and the radial or normal force produces radial deflection of the rim and bending of the shafts.

**Effect of Altering the Centre Distance on the Velocity Ratio for Involute Teeth Gears:**

In the previous article, we have seen that the velocity ratio for the involute teeth gears is given by

\[
\frac{Q_1M}{Q_1N} = \frac{Q_1P}{Q_1P} = \frac{o_2}{o_1}
\]

Let, in Fig. 12.10 (a), the centre of rotation of one of the gears (say wheel 1) is shifted from \( O_1 \) to \( O_1' \). Consequently the contact point shifts from \( Q \) to \( Q' \).
common normal to the teeth at the point of contact $Q'$ is the tangent to the base circle, because it has a contact between two involute curves and they are generated from the base circle. Let the tangent $M'N'$ to the base circles intersects $O_1'O_2$ at the pitch point $P'$. As a result of this, the wheel continues to work correctly.

Now from similar triangles $O_2NP$ and $O_1MP$,

$$\frac{QM}{O_2N} = \frac{O_1P}{O_1P} \quad \text{...(ii)}$$

and from similar triangles $O_2N'P'$ and $O_1'M'P'$,

$$\frac{Q'M'}{O_2N'} = \frac{Q'I'}{O_1'P'} \quad \text{...(iii)}$$

But $O_2N = O_2N'$, and $O_1M = O_1'M'$. Therefore from equations (ii) and (iii),

$$\frac{O_1P}{O_1P} = \frac{O_1'P'}{O_1'P'} \quad \text{...[Same as equation (ii)]}$$

Thus we see that if the centre distance is changed within limits, the velocity ratio remains unchanged. However, the pressure angle increases (from $\phi$ to $\phi'$) with the increase in the centre distance.

**Example:** A single reduction gear of 120 kW with a pinion 250 mm pitch circle diameter and speed 650 r.p.m. is supported in bearings on either side. Calculate the total load due to the power transmitted, the pressure angle being 20°.

**Solution:** Given: $P = 120 \text{ kW} = 120 \times 10^3 \text{ W}$; $d = 250 \text{ mm}$; $N = 650 \text{ r.p.m.}$ or $\omega = 2\pi \times 650/60 = 68 \text{ rad/s}$; $\phi = 20^\circ$

Let $T = \text{Torque transmitted in N-m.}$

We know that power transmitted $(P)\quad 120 \times 103 = T \cdot \omega = T \times 68$ or $T = 120 \times 103/68 = 1765 \text{ N-m}$

and tangential load on the pinion, $F_T = T/r = 1765 / 0.125 = 14 120 \text{ N}$

$\therefore$ Total load due to power transmitted,

$$F = F_T / \cos \phi = 14 120 / \cos 20^\circ = 15 026 \text{ N} = 15.026 \text{ kN} \quad \text{Ans.}$$

**Systems of Gear Teeth:**

The following four systems of gear teeth are commonly used in practice:

1. **14.5° Composite system**,  
2. **14.5° Full depth involute system**,  
3. **20° Full depth involute system**, and  
4. **20° Stub involute system**.

The **14.5° composite system** is used for general purpose gears. It is stronger but has no interchangeability. The tooth profile of this system has cycloidal curves at the top and bottom and involute curve at the middle portion. The teeth are produced by formed milling cutters or hobs. The tooth profile of the **14.5° full depth involute system** was developed for use with gear hobs for spur and helical gears.

The tooth profile of the **20° full depth involute system** may be cut by hobs. The increase of the pressure angle from 14.5° to 20° results in a stronger tooth, because the tooth acting as a beam is wider at the base. The **20° stub involute system** has a strong tooth to take heavy loads.

**Standard Proportions of Gear Systems:**
The following table shows the standard proportions in module \((m)\) for the four gear systems as discussed in the previous article.

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Particulars</th>
<th>(14\frac{1}{2})° composite or full depth involute system</th>
<th>20° full depth involute system</th>
<th>20° stub involute system</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Addendum</td>
<td>1 (m)</td>
<td>1 (m)</td>
<td>0.8 (m)</td>
</tr>
<tr>
<td>2.</td>
<td>Dedendum</td>
<td>1.25 (m)</td>
<td>1.25 (m)</td>
<td>1 (m)</td>
</tr>
<tr>
<td>3.</td>
<td>Working depth</td>
<td>2 (m)</td>
<td>2 (m)</td>
<td>1.60 (m)</td>
</tr>
<tr>
<td>4.</td>
<td>Minimum total depth</td>
<td>2.25 (m)</td>
<td>2.25 (m)</td>
<td>1.80 (m)</td>
</tr>
<tr>
<td>5.</td>
<td>Tooth thickness</td>
<td>1.5708 (m)</td>
<td>1.5708 (m)</td>
<td>1.5708 (m)</td>
</tr>
<tr>
<td>6.</td>
<td>Minimum clearance</td>
<td>0.25 (m)</td>
<td>0.25 (m)</td>
<td>0.2 (m)</td>
</tr>
<tr>
<td>7.</td>
<td>Fillet radius at root</td>
<td>0.4 (m)</td>
<td>0.4 (m)</td>
<td>0.4 (m)</td>
</tr>
</tbody>
</table>

**Length of Path of Contact:**

Consider a pinion driving the wheel as shown in Fig. 12.11. When the pinion rotates in clockwise direction, the contact between a pair of involute teeth begins at \(K\) (on the flank near the base circle of pinion or the outer end of the tooth face on the wheel) and \((If\ the\ wheel\ is\ made\ to\ act\ as\ a\ driver\ and\ the\ directions\ of\ motion\ are\ reversed,\ then\ the\ contact\ between\ a\ pair\ of\ teeth\ begins\ at\ L\ and\ ends\ at\ K.)\) ends at \(L\) (outer end of the tooth face on the pinion or on the flank near the base circle of wheel). \(MN\) is the common normal at the point of contacts and the common tangent to the base circles. The point \(K\) is the intersection of the addendum circle of wheel and the common tangent. The point \(L\) is the intersection of the addendum circle of pinion and common tangent.

![Fig. 12.11. Length of path of contact.](image)

We have discussed previously that the length of path of contact is the length of common normal cut off by the addendum circles of the wheel and the pinion. Thus the length of path of contact is \(KL\) which is the sum of the parts of the path of contacts \(KP\) and \(PL\). The part of the path of contact \(KP\) is known as *path of approach* and the part of the path of contact \(PL\) is known as *path of recess*.

Let \(r_A = O_1L = \text{Radius of addendum circle of pinion},\)

\(R_A = O_2K = \text{Radius of addendum circle of wheel},\)
Toothed Gearing

\[ r = O_1 P = \text{Radius of pitch circle of pinion, and} \]
\[ R = O_2 P = \text{Radius of pitch circle of wheel.} \]
From Fig. 12.11, we find that radius of the base circle of pinion,
\[ O_1 M = O_1 P \cos \phi = r \cos \phi \]
and radius of the base circle of wheel,
\[ O_2 N = O_2 P \cos \phi = R \cos \phi \]
Now from right angled triangle \( O_1K \),
\[ KN = \sqrt{(O_2 K)^2 - (O_2 M)^2} = \sqrt{(R_1)^2 - r^2 \cos^2 \phi} \]
and \( PN = O_2 P \sin \phi = R \sin \phi \)
\[ KP = KN - PN = \sqrt{(R_1)^2 - r^2 \cos^2 \phi} - R \sin \phi \]
Similarly from right angled triangle \( O_1M \),
\[ ML = \sqrt{(O_1 L)^2 - (O_1 M)^2} = \sqrt{(r_1)^2 - r^2 \cos^2 \phi} \]
\[ MP = O_1 P \sin \phi = r \sin \phi \]
\[ PL = ML - MP = \sqrt{(r_1)^2 - r^2 \cos^2 \phi} - r \sin \phi \]
\[ KL = KP + PL = \sqrt{(R_1)^2 - r^2 \cos^2 \phi} + \sqrt{(r_1)^2 - r^2 \cos^2 \phi} - (R + r) \sin \phi \]

**Length of Arc of Contact:**

We have already defined that the arc of contact is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth. In Fig. 12.11, the arc of contact is \( EPF \) or \( GPH \). Considering the arc of contact \( GPH \), it is divided into two parts i.e. arc \( GP \) and arc \( PH \). The arc \( GP \) is known as arc of approach and the arc \( PH \) is called arc of recess. The angles subtended by these arcs at \( O_1 \) are called angle of approach and angle of recess respectively.

We know that the length of the arc of approach (arc \( GP \))
\[ \frac{\text{Length of path of approach}}{\cos \phi} = \frac{KP}{\cos \phi} \]
and the length of the arc of recess (arc \( PH \))
\[ \frac{\text{Length of path of recess}}{\cos \phi} = \frac{PL}{\cos \phi} \]
Since the length of the arc of contact \( GPH \) is equal to the sum of the length of arc of approach and arc of recess, therefore,
Length of the arc of contact
\[ = \text{arc } GP + \text{arc } PH = \frac{KP}{\cos \phi} + \frac{PL}{\cos \phi} = \frac{KL}{\cos \phi} \]
The contact ratio or the number of pairs of teeth in contact is defined as the ratio of the length of the arc of contact to the circular pitch. Mathematically,
Contact ratio or number of pairs of teeth in contact
where \( p_c = \text{Circular pitch} = \pi m \), and \( m = \text{Module} \).

**Notes:**
1. The contact ratio, usually, is not a whole number. For example, if the contact ratio is 1.6, it does not mean that there are 1.6 pairs of teeth in contact. It means that there are alternately one pair and two pairs of teeth in contact and on a time basis the average is 1.6.
2. The theoretical minimum value for the contact ratio is one, that is there must always be at least one pair of teeth in contact for continuous action.
3. Larger the contact ratio, more quietly the gears will operate.

**Example:** The number of teeth on each of the two equal spur gears in mesh are 40. The teeth have 20° involute profile and the module is 6 mm. If the arc of contact is 1.75 times the circular pitch, find the addendum.

**Solution:**
Given:
\( T = t = 40 \); \( \phi = 20^\circ \); \( m = 6 \text{ mm} \)

We know that the circular pitch,
\( p_c = \pi m = \pi \times 6 = 18.85 \text{ mm} \)

\[ p_c = 1.75 \times 18.85 = 33 \text{ mm} \]

and length of path of contact = Length of arc of contact \( \times \cos \phi \)
\[ = 33 \cos 20^\circ = 31 \text{ mm} \]

Let \( R_A = r_A = \text{Radius of the addendum circle of each wheel} \).

We know that pitch circle radii of each wheel,
\( R = r = m.T / 2 = 6 \times 40/2 = 120 \text{ mm} \)

and length of path of contact
\[ 31 = \sqrt{(R_A)^2 - (r_A)^2 \cos^2 \phi} + \sqrt{(r_A)^2 - r^2 \cos^2 \phi} - (R + r) \sin \phi \]

\[ = 2 \left[ \sqrt{(R_A)^2 - (R^2 \cos^2 \phi)} - R \sin \phi \right] \ldots(\because R = r, \text{ and } R_A = r_A) \]

\[ \frac{31}{2} = \sqrt{(R_A)^2 - (120)^2 \cos^2 20^\circ} - 120 \sin 20^\circ \]

\[ 15.5 = \sqrt{(R_A)^2 - 12715} = 41 \]

\[ (15.5 + 41)^2 = (R_A)^2 - 12715 \]

\[ 3192 + 12715 = (R_A)^2 \text{ or } R_A = 126.12 \text{ mm} \]

We know that the addendum of the wheel,
\( = R_A - R = 126.12 - 120 = 6.12 \text{ mm} \) **Ans.**

**Example:** A pinion having 30 teeth drives a gear having 80 teeth. The profile of the gears is involute with 20° pressure angle, 12 mm module and 10 mm addendum. Find the length of path of contact, arc of contact and the contact ratio.

**Solution:**
Given:
\( t = 30; T = 80; \phi = 20^\circ; m = 12 \text{ mm}; \text{Addendum} = 10 \text{ mm} \)

**Length of path of contact**

We know that pitch circle radius of pinion, \( r = m.t / 2 = 12 \times 30 / 2 = 180 \text{ mm} \)

and pitch circle radius of gear, \( R = m.T / 2 = 12 \times 80 / 2 = 480 \text{ mm} \)

\( \therefore \) Radius of addendum circle of pinion, \( r_A = r + \text{Addendum} = 180 + 10 = 190 \text{ mm} \)

and radius of addendum circle of gear, \( R_A = R + \text{Addendum} = 480 + 10 = 490 \text{ mm} \)

We know that length of the path of approach,
Toothed Gearing

\[ KP = \sqrt{\left(R_A \right)^2 - R^2 \cos^2 \phi - R \sin \phi} \]  

... (Refer Fig. 12.11)

and length of the path of recess,

\[ PL = \sqrt{\left(r_A \right)^2 - r^2 \cos^2 \phi - r \sin \phi} \]

We know that length of path of contact, \( KL = KP + PL = 27.3 + 25 = 52.3 \text{ mm} \) \( \text{Ans.} \)

**Length of arc of contact**

We know that length of arc of contact

\[ \text{Length of arc of contact} = \frac{52.3}{\cos 20^\circ} = 55.66 \text{ mm} \] \( \text{Ans.} \)

**Contact ratio**

We know that circular pitch, \( p_c = \pi m = \pi \times 12 = 37.7 \text{ mm} \)

\[ \therefore \text{Contact ratio} = \frac{\text{Length of arc of contact}}{\frac{p_c}{\cos 20^\circ}} = \frac{55.66}{37.7} = 1.5 \text{ say 2} \] \( \text{Ans.} \)

**Example:** Two involute gears of 20\(^\circ\) pressure angle are in mesh. The number of teeth on pinion is 20 and the gear ratio is 2. If the pitch expressed in module is 5 mm and the pitch line speed is 1.2 m/s, assuming addendum as standard and equal to one module, find:

1. The angle turned through by pinion when one pair of teeth is in mesh; and
2. The maximum velocity of sliding.

**Solution:** Given: \( \phi = 20^\circ; \ t = 20; \ G = T/t = 2; \ m = 5 \text{ mm}; \ v = 1.2 \text{ m/s}; \) addendum = 1 module= 5 mm

1. **Angle turned through by pinion when one pair of teeth is in mesh**

   We know that pitch circle radius of pinion, \( r = m.t / 2 = 5 \times 20 / 2 = 50 \text{ mm} \) and pitch circle radius of wheel,

   \[ R = m.T / 2 = m.G.t / 2 = 2 \times 20 \times 5 / 2 = 100 \text{ mm} \]  

   \( \therefore \) Radius of addendum circle of pinion, \( r_A = r + \text{Addendum} = 50 + 5 = 55 \text{ mm} \) and radius of addendum circle of wheel, \( R_A = R + \text{Addendum} = 100 + 5 = 105 \text{ mm} \)

   We know that length of the path of approach (i.e. the path of contact when engagement occurs),

   \[ KP = \sqrt{\left(R_A \right)^2 - R^2 \cos^2 \phi - R \sin \phi} \]  

   \( = \sqrt{(105)^2 - (100)^2 \cos 20^\circ - 100 \sin 20^\circ} \)

   \( = 46.85 - 34.2 = 12.65 \text{ mm} \)

   and the length of path of recess (i.e. the path of contact when disengagement occurs),

   \[ PL = \sqrt{\left(r_A \right)^2 - r^2 \cos^2 \phi - r \sin \phi} \]

   \( = \sqrt{(55)^2 - (50)^2 \cos 20^\circ - 50 \sin 20^\circ} = 28.6 - 17.1 = 11.5 \text{ mm} \)

   \( \therefore \) Length of the path of contact, \( KL = KP + PL = 12.65 + 11.5 = 24.15 \text{ mm} \) and length of the arc of contact
We know that angle turned through by pinion
\[
\frac{\text{Length of arc of contact} \times 360^\circ}{\text{Circumference of pinion}} = \frac{25.7 \times 360^\circ}{2\pi \times 50} = 29.45^\circ \quad \text{Ans.}
\]

2. Maximum velocity of sliding
Let \( \omega_1 = \text{Angular speed of pinion} \) and \( \omega_2 = \text{Angular speed of wheel} \).
We know that pitch line speed,
\[
v = \omega_1 r = \omega_2 R
\]
\[
\therefore \omega_1 = v/r = 120/5 = 24 \text{ rad/s} \quad \text{and} \quad \omega_2 = v/R = 120/10 = 12 \text{ rad/s}
\]
\[
\therefore \text{Maximum velocity of sliding},
v_S = (\omega_1 + \omega_2) K_P \quad \text{...(K_P > P_L)}
\]
\[
v_S = (24 + 12) 12.65 = 455.4 \text{ mm/s} \quad \text{Ans.}
\]

Example: A pair of gears, having 40 and 20 teeth respectively, are rotating in mesh, the speed of the smaller being 2000 r.p.m. Determine the velocity of sliding between the gear teeth faces at the point of engagement, at the pitch point, and at the point of disengagement if the smaller gear is the driver. Assume that the gear teeth are 20° involute form, addendum length is 5 mm and the module is 5 mm.
Also find the angle through which the pinion turns while any pairs of teeth are in contact.

Solution: Given : \( T = 40 \), \( t = 20 \), \( N_1 = 2000 \text{ r.p.m.} \), \( \phi = 20^\circ \), addendum = 5 mm ; \( m = 5 \text{ mm} \)
We know that angular velocity of the smaller gear,
\[
\omega_2 = \frac{2\pi N_1}{60} = \frac{2\pi \times 2000}{60} = 209.5 \text{ rad/s}
\]
and angular velocity of the larger gear,
\[
\omega_1 = \frac{t}{T} \times \frac{209.5}{40} = 104.75 \text{ rad/s}
\]
Pitch circle radius of the smaller gear, \( r = m.t / 2 = 5 \times 20/2 = 50 \text{ mm} \)
and pitch circle radius of the larger gear, \( R = m.t / 2 = 5 \times 40/2 = 100 \text{ mm} \)
\[
\therefore \text{Radius of addendum circle of smaller gear} = r + \text{Addendum} = 50 + 5 = 55 \text{ mm}
\]
\[
\text{and radius of addendum circle of larger gear} = R + \text{Addendum} = 100 + 5 = 105 \text{ mm}
\]
The engagement and disengagement of the gear teeth is shown in Fig. 12.11. The point \( K \) is the point of engagement, \( P \) is the pitch point and \( L \) is the point of disengagement. \( MN \) is the common tangent at the points of contact.

Interference in Involute Gears:
Fig. 12.13 shows a pinion with centre \( O_1 \), in mesh with wheel or gear with centre \( O_2 \). \( MN \) is the common tangent to the base circles and \( KL \) is the path of contact between the two mating teeth. A little consideration will show, that if the radius of the addendum circle of pinion is increased to \( O_1N \), the point of contact \( L \) will move from \( L \) to \( N \). When this radius is further increased, the point of contact \( L \) will be on the inside of base circle of wheel and not on the involute profile of tooth on wheel. The tip of tooth on the pinion will then undercut the tooth on the wheel at the root and remove part of the involute profile of tooth on the wheel. This effect is known as
interference, and occurs when the teeth are being cut. In brief, the phenomenon when the tip of tooth undercuts the root on its mating gear is known as interference.

Similarly, if the radius of the addendum circle of the wheel increases beyond $O_2M$, then the tip of tooth on wheel will cause interference with the tooth on pinion. The points $M$ and $N$ are called interference points. Obviously, interference may be avoided if the path of contact does not extend beyond interference points. The limiting value of the radius of the addendum circle of the pinion is $O_1N$ and of the wheel is $O_2M$.

From the above discussion, we conclude that the interference may only be avoided, if the point of contact between the two teeth is always on the involute profiles of both the teeth. In other words, interference may only be prevented, if the addendum circles of the two mating gears cut the common tangent to the base circles between the points of tangency.

![Fig. 12.13. Interference in involute gears.](image)

When interference is just avoided, the maximum length of path of contact is $MN$ when the maximum addendum circles for pinion and wheel pass through the points of tangency $N$ and $M$ respectively as shown in Fig. 12.13. In such a case,

Maximum length of path of approach, $MP = r \sin \phi$
and maximum length of path of recess, $PN = R \sin \phi$

$\therefore$ Maximum length of path of contact,

$MN = MP + PN = r \sin \phi + R \sin \phi = (r + R) \sin \phi$

and maximum length of arc of contact

$$= \frac{(r + R) \sin \phi}{\cos \phi} = (r + R) \tan \phi$$

**Note:** In case the addenda on pinion and wheel is such that the path of approach and path of recess are half of their maximum possible values, then

**Path of approach, $KP = 0.5 \, MP$**

or

$$\sqrt{(r_1)^2 - r^2 \cos^2 \phi} - r \sin \phi = \frac{r \sin \phi}{2}$$

and path of recess,

$$PL = \frac{1}{2} \, PN$$

or

$$\sqrt{(r_1)^2 - r^2 \cos^2 \phi} - r \sin \phi = \frac{R \sin \phi}{2}$$

$\therefore$ Length of the path of contact
Toothed Gearing

Minimum Number of Teeth on the Pinion in Order to Avoid Interference:

We have already discussed in the previous article that in order to avoid interference, the addendum circles for the two mating gears must cut the common tangent to the base circles between the points of tangency. The limiting condition reaches, when the addendum circles of pinion and wheel pass through points $N$ and $M$ (see Fig. 12.13) respectively.

Let $t =$ Number of teeth on the pinion,

$T =$ Number of teeth on the wheel,

$m =$ Module of the teeth,

$r =$ Pitch circle radius of pinion $= mt / 2$

$G =$ Gear ratio $= T / t = R / r$

$\phi =$ Pressure angle or angle of obliquity.

From triangle $O_1NP$,

$$\begin{align*}
(O_1N)^2 &= (O_1P)^2 + (PN)^2 - 2 \times O_1P \times PN \cos O_1PN \\
&= r^2 + R^2 \sin^2 \phi - 2r \cdot R \sin \phi \cos (90^\circ + \phi)
\end{align*}$$

...(2) $PN = O_2P \sin \phi = R \sin \phi$

$$\begin{align*}
&= r^2 + R^2 \sin^2 \phi + 2r \cdot R \sin^2 \phi \\
&= r^2 \left[ 1 + \frac{R^2 \sin^2 \phi}{r^2} + 2 \frac{R \sin^2 \phi}{r} \right] = r^2 \left[ 1 + \frac{R}{r} \left( \frac{R}{r} + 2 \right) \sin^2 \phi \right]
\end{align*}$$

.: Limiting radius of the pinion addendum circle,

$$O_1N = r \sqrt{1 + \frac{R}{r} \left( \frac{R}{r} + 2 \right) \sin^2 \phi} = \frac{mt}{2} \sqrt{1 + \frac{T}{t} \left( \frac{T}{t} + 2 \right) \sin^2 \phi}$$

Let $A_P.m =$ Addendum of the pinion, where $A_P$ is a fraction by which the standard addendum of one module for the pinion should be multiplied in order to avoid interference.

We know that the addendum of the pinion $= O_1N - O_1P$

$$\begin{align*}
A_P.m &= \frac{mt}{2} \sqrt{1 + \frac{T}{t} \left( \frac{T}{t} + 2 \right) \sin^2 \phi} - \frac{mt}{2} \\
&= \frac{mt}{2} \left[ 1 + \frac{\frac{T}{t} \left( \frac{T}{t} + 2 \right) \sin^2 \phi}{-1} \right]
\end{align*}$$

$$A_P = \frac{t}{2} \left[ 1 + \frac{\frac{T}{t} \left( \frac{T}{t} + 2 \right) \sin^2 \phi}{-1} \right]$$

$$\therefore \quad r = \frac{2 A_P}{\sqrt{1 + \frac{T}{t} \left( \frac{T}{t} + 2 \right) \sin^2 \phi} - 1} = \frac{2 A_P}{\sqrt{1 + G \left( G + 2 \right) \sin^2 \phi} - 1}$$

This equation gives the minimum number of teeth required on the pinion in order to avoid interference.
Notes: 1. If the pinion and wheel have equal teeth, then \( G = 1 \). Therefore the above equation reduces to

\[
T = \frac{2A_p}{\sqrt{1 + 3 \sin^2 \phi - 1}}
\]

2. The minimum number of teeth on the pinion which will mesh with any gear (also rack) without interference are given in the following table:

<table>
<thead>
<tr>
<th>S. No.</th>
<th>System of gear teeth</th>
<th>Minimum number of teeth on the pinion</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>143.5 Composite</td>
<td>12</td>
</tr>
<tr>
<td>2.</td>
<td>143.5 Full depth involute</td>
<td>32</td>
</tr>
<tr>
<td>3.</td>
<td>20° Full depth involute</td>
<td>18</td>
</tr>
<tr>
<td>4.</td>
<td>20° Stub involute</td>
<td>14</td>
</tr>
</tbody>
</table>

Minimum Number of Teeth on the Wheel in Order to Avoid Interference:

Let \( T = \) Minimum number of teeth required on the wheel in order to avoid interference, and \( A_{W,m} = \) Addendum of the wheel, where \( AW \) is a fraction by which the standard addendum for the wheel should be multiplied.

Using the same notations as in previous section, we have from triangle \( O_2MP \)

\[
(O_2M)^2 = (O_2P)^2 + (PM)^2 - 2 \times O_2P \times PM \cos O_2PM
\]

\[
= R^2 + r^2 \sin^2 \phi - 2r \sin \phi \cos (90^\circ + \phi)
\]

\[
= R^2 + r^2 \sin^2 \phi + 2r \sin \phi
\]

\[
= R^2 \left[ 1 + \frac{r^2 \sin^2 \phi}{R^2} + \frac{2r \sin \phi}{R} \right] = R^2 \left[ 1 + \frac{R}{R} \left( \frac{r}{R} + 2 \right) \sin^2 \phi \right]
\]

\[\therefore \text{ Limiting radius of wheel addendum circle,} \]

\[
O_2M = R \sqrt{1 + \frac{R}{R} \left( \frac{r}{R} + 2 \right) \sin^2 \phi} = \frac{mT}{2} \sqrt{1 + \frac{t}{T} \left( \frac{t}{T} + 2 \right) \sin^2 \phi}
\]

We know that the addendum of the wheel

\[
O_2M = O_2P - O_2P
\]

\[\therefore A_{W,m} = \frac{mT}{2} \sqrt{1 + \frac{t}{T} \left( \frac{t}{T} + 2 \right) \sin^2 \phi} - \frac{mT}{2}
\]

\[\therefore A_{W,m} = \frac{mT}{2} \left[ \sqrt{1 + \frac{t}{T} \left( \frac{t}{T} + 2 \right) \sin^2 \phi} - 1 \right]
\]

or

\[
A_{W} = \frac{T}{2} \sqrt{1 + \frac{t}{T} \left( \frac{t}{T} + 2 \right) \sin^2 \phi} - 1
\]

\[\therefore T = \frac{2A_W}{\sqrt{1 + \frac{t}{T} \left( \frac{t}{T} + 2 \right) \sin^2 \phi} - 1} = \frac{2A_W}{\sqrt{1 + \frac{1}{G} \left( \frac{1}{G} + 2 \right) \sin^2 \phi} - 1}
\]

Notes: 1. From the above equation, we may also obtain the minimum number of teeth on pinion. Multiplying both sides by \((t/T)\),
2. If wheel and pinion have equal teeth, then \( G = 1 \), and

\[
T = \frac{2A_W}{\sqrt{1 + 3 \sin^2 \phi - 1}}
\]

**Example:** Determine the minimum number of teeth required on a pinion, in order to avoid interference which is to gear with, 1. a wheel to give a gear ratio of 3 to 1; and 2. an equal wheel. The pressure angle is 20° and a standard addendum of 1 module for the wheel may be assumed.

**Solution:** Given: \( G = T / t = 3; \phi = 20^{\circ}; A_W = 1 \) module

1. **Minimum number of teeth for a gear ratio of 3 : 1**

We know that minimum number of teeth required on a pinion,

\[
t = \frac{2 \times A_W}{G \left[ 1 + \frac{1}{G} \left( \frac{1}{G} + 2 \right) \sin^2 \phi - 1 \right]}
\]

\[
t = \frac{2 \times 1}{\frac{2}{0.133}} = 15.04 \text{ or } 16 \quad \text{Ans.}
\]

2. **Minimum number of teeth for equal wheel**

We know that minimum number of teeth for equal wheel,

\[
t = \frac{2 \times A_W}{\sqrt{1 + 3 \sin^2 \phi - 1}} = \frac{2 \times 1}{\sqrt{1 + 3 \sin^2 20^\circ - 1}} = \frac{2}{0.162}
\]

\[
= 12.34 \text{ or } 13 \quad \text{Ans.}
\]

**Helical Gears:**

A helical gear has teeth in the form of helix around the gear. Two such gears may be used to connect two parallel shafts in place of spur gear. The helixes may be right handed on one wheel and left handed on the other. The pitch surfaces are cylindrical as in spur gearing, but the teeth instead of being parallel to the axis, wind around the cylinders helically like screw threads. The teeth of helical gears with parallel axis have line contact, as in spur gearing. This provides gradual engagement and continuous contact of the engaging teeth. Hence helical gears give smooth drive with a high efficiency of transmission.

We have already discussed that the helical gears may be of single helical type or double helical type. In case of single helical gears, there is some axial thrust between the teeth, which is a disadvantage. In order to eliminate this axial thrust, double
helical gears are used. It is equivalent to two single helical gears, in which equal and opposite thrusts are produced on each gear and the resulting axial thrust is zero.

The following definitions may be clearly understood in connection with a helical gear as shown in Fig. 12.15.

1. Normal pitch. It is the distance between similar faces of adjacent teeth, along a helix on the pitch cylinder normal to the teeth. It is denoted by \( p_N \).
2. Axial pitch. It is the distance measured parallel to the axis, between similar faces of adjacent teeth. It is the same as circular pitch and is therefore denoted by \( p_c \). If \( \alpha \) is the helix (or spiral) angle, then circular pitch,

\[
\rho_c = \frac{p_N}{\cos \alpha}
\]

**Spiral Gears:**

We have already discussed that spiral gears (also known as skew gears or screw gears) are used to connect and transmit motion between two non-parallel and non-intersecting shafts. The pitch surfaces of the spiral gears are cylindrical and the teeth have point contact. These gears are only suitable for transmitting small power. We have seen that helical gears, connected on parallel shafts, are of opposite hand. But spiral gears may be of the same hand or of opposite hand.

**Centre Distance for a Pair of Spiral Gears:**

The centre distance, for a pair of spiral gears, is the shortest distance between the two shafts making any angle between them. A pair of spiral gears 1 and 2, both having left hand helixes (i.e. the gears are of the same hand) is shown in Fig. 12.16. The shaft angle \( \theta \) is the angle through which one of the shafts must be rotated so that it is parallel to the other shaft, also the two shafts be rotating in opposite directions.

Let \( \alpha_1 \) and \( \alpha_2 \) = Spiral angles of gear teeth for gears 1 and 2 respectively,
\( p_{c1} \) and \( p_{c2} \) = Circular pitches of gears 1 and 2,
\( T_1 \) and \( T_2 \) = Number of teeth on gears 1 and 2,
\( d_1 \) and \( d_2 \) = Pitch circle diameters of gears 1 and 2,
\( N_1 \) and \( N_2 \) = Speed of gears 1 and 2,
\( G = \text{Gear ratio} = \frac{T_2}{T_1} = \frac{N_1}{N_2} \),
\( p_N \) = Normal pitch, and \( L = \) Least centre distance between the axes of shafts.

Since the normal pitch is same for both the spiral gears, therefore
Notes: 1. If the pair of spiral gears have teeth of the same hand, then \[ \theta = \alpha_1 + \alpha_2 \] and for a pair of spiral gears of opposite hand, \[ \theta = \alpha_1 - \alpha_2. \]
2. When \( \theta = 90^\circ \), then both the spiral gears must have teeth of the same hand.

Efficiency of Spiral Gears:
A pair of spiral gears 1 and 2 in mesh is shown in Fig. 12.17. Let the gear 1 be the driver and the gear 2 the driven. The forces acting on each of a pair of teeth in contact are shown in Fig. 12.17. The forces are assumed to act at the centre of the width of each tooth and in the plane tangential to the pitch cylinders.

Let
- \( F_1 \) = Force applied tangentially on the driver,
- \( F_2 \) = Resisting force acting tangentially on the driven,
- \( F_{a1} \) = Axial or end thrust on the driver,
- \( F_{a2} \) = Axial or end thrust on the driven,
- \( R_N \) = Normal reaction at the point of contact,
- \( \phi \) = Angle of friction,
\[ R = \text{Resultant reaction at the point of contact, and} \]
\[ \theta = \text{Shaft angle} = \alpha_1 + \alpha_2 \quad \text{...(Both gears are of the same hand)} \]

**Fig. 12.17. Efficiency of spiral gears.**

From triangle \(OPQ\), \(F_1 = R \cos (\alpha_1 - \phi)\)

\[ \therefore \text{Work input to the driver} = F_1 \times \pi \cdot d_1 \cdot N_1 = R \cos (\alpha_1 - \phi) \pi \cdot d_1 \cdot N_1 \]

From triangle \(OST\), \(F_2 = R \cos (\alpha_2 - \phi)\)

\[ \therefore \text{Work output of the driven} = F_2 \times \pi \cdot d_2 \cdot N_2 = R \cos (\alpha_2 - \phi) \pi \cdot d_2 \cdot N_2 \]

\[ \therefore \text{Efficiency of spiral gears,} \]

\[ \eta = \frac{\text{Work output}}{\text{Work input}} = \frac{R \cos (\alpha_2 + \phi) \pi \cdot d_2 \cdot N_2}{R \cos (\alpha_1 - \phi) \pi \cdot d_1 \cdot N_1} \]

We have discussed in previous sections, that pitch circle diameter of gear 1,

\[ d_1 = \frac{P_1 \times T_1}{\pi} = \frac{R_1}{\cos \alpha_1} \times \frac{T_1}{\pi} \]

and pitch circle diameter of gear 2,

\[ d_2 = \frac{P_2 \times T_2}{\pi} = \frac{R_2}{\cos \alpha_2} \times \frac{T_2}{\pi} \]

\[ \therefore \quad \frac{d_1}{d_2} = \frac{\cos \alpha_1}{\cos \alpha_2} \quad \text{...(ii)} \]

We know that \(\frac{N_2}{N_1} = \frac{T_1}{T_2} \quad \text{...(iii)}\)

Multiplying equations (ii) and (iii), we get,

\[ \frac{d_2 \cdot N_2}{d_1 \cdot N_1} = \frac{\cos \alpha_1}{\cos \alpha_2} \]

Substituting this value in equation (i), we have
Since the angles \( \theta \) and \( \phi \) are constants, therefore the efficiency will be maximum, when
\[
\cos (\alpha_1 - \alpha_2 - \phi) \text{ is maximum, i.e. } \cos (\alpha_1 - \alpha_2 - \phi) = 1 \text{ or } \alpha_1 - \alpha_2 - \phi = 0
\]

Thus \( \alpha_1 = \alpha_2 + \phi \) and \( \alpha_2 = \alpha_1 - \phi \)

Since \( \alpha_1 + \alpha_2 = 0 \), therefore

\[
\alpha_1 = \frac{\theta - \alpha_2}{2} \quad \text{or} \quad \alpha_1 = \frac{\theta + \phi}{2}
\]

Similarly,

\[
\alpha_2 = \frac{\theta - \phi}{2}
\]

Substituting \( \alpha_1 = \alpha_2 + \phi \) and \( \alpha_2 = \alpha_1 - \phi \), in equation (v), we get

\[
\eta_{\text{max}} = \frac{\cos (\theta + \phi) + 1}{\cos (\theta - \phi) + 1}
\]

**Note:** From Fig. 12.17, we find that

\[
R_n = \frac{F_1}{\cos \alpha_1} = \frac{F_2}{\cos \alpha_2}
\]

Axial thrust on the driver, \( F_{a1} = R_n \sin \alpha_1 = F_2 \tan \alpha_1 \)

and axial thrust on the driven, \( F_{a2} = R_n \sin \alpha_2 = F_2 \tan \alpha_2 \)

**Example:** A pair of spiral gears is required to connect two shafts 175 mm apart, the shaft angle being 70°. The velocity ratio is to be 1.5 to 1, the faster wheel having 80 teeth and a pitch circle diameter of 100 mm. Find the spiral angles for each wheel.

If the torque on the faster wheel is 75 N-m ; find the axial thrust on each shaft, neglecting friction.

**Solution:**

Given : \( L = 175 \text{ mm} = 0.175 \text{ m} \); \( \theta = 70^\circ \); \( G = 1.5 \); \( T_2 = 80 \); \( d_2 = 100 \text{ mm} \) or \( r_2 = 0.05 \text{ m} \); Torque on faster wheel = 75 N-m

**Spiral angles for each wheel**

Let \( \alpha_1 \) = Spiral angle for slower wheel, and \( \alpha_2 \) = Spiral angle for faster wheel.

We know that velocity ratio,

\[
G = \frac{N_2}{N_1} = \frac{T_1}{T_2} = 1.5
\]

Thus No. of teeth on slower wheel, \( T_1 = 80 \times 1.5 = 120 \)

We also know that the centre distance between shafts (L),
Toothed Gearing

Axial thrust on each shaft

We know that Torque = Tangential force × Pitch circle radius

\[
F_2 = \frac{\text{Torque on the faster wheel}}{\text{Pitch circle radius (} \ell_2 \text{)}} = \frac{75}{0.05} = 1500 \text{ N}
\]

and normal reaction at the point of contact,

\[R_N = F_2 \cos \alpha_2 = 1500 \cos 15.35^\circ = 1556 \text{ N}\]

We know that axial thrust on the shaft of slower wheel,

\[F_{a1} = R_N \sin \alpha_1 = 1556 \times \sin 54.65^\circ = 1269 \text{ N \hspace{1em} Ans.}\]

and axial thrust on the shaft of faster wheel,

\[F_{a2} = R_N \sin \alpha_2 = 1556 \times \sin 15.35^\circ = 412 \text{ N \hspace{1em} Ans.}\]